RUHR-UNIVERSITÄT BOCHUM

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Using randomized quizzes in undergraduate linear algebra and multivariable calculus

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Overview

1 Setting

2 Randomization

3 Use by students

Typical situation

Mathematics Course

- Lectures (2 x 90 min.)
- Exercise class (90 min.) as a preparation for
- homework exercises

- large undergraduate courses (100-600 students)
- take place in a similar way every year
- set of exercises often already available
- bonus points as an incentive

Transition to computer-aided assessment

Motivation for transition

- less marking of routine exercises
- remove opportunity to copy from old or friends' notes
- lecturer can get feedback on learning results

Classify exercises

Suitable without modification	Not suitable for electronic assessment	electronic version superior
Mostly routine calculations	e.g. reasoning/proofs	e.g. giving examples, exercises with many solutions

Implementation

Mixture of exercise types

Replace **4 written homework exercises**

by **3 written homework exercises + 1 STACK exercise**

or **2 written homework exercises + 2 STACK exercises**

– Mathematics for physicists 1-3 (1 STACK exercise since 2015/16)
– Analysis 1 (1 STACK exercise)
– Linear algebra 1 (1 STACK exercise)
– Mathematics for engineers (1-2 STACK exercises)
– Mathematics for chemists (varying number of STACK exercises)

Implementation

Goals

- 1 STACK exercise per week, i.e. STACK exercises for each topic
- try to randomize exercises
- allow for multiple attempts $\rightsquigarrow$ low stakes

Challenges/limits

- Some topics more difficult (metric spaces, measure theory,...)
- Randomization must offer a sufficiently large variety of problems

Topics in Linear Algebra

Examples

- systems of linear equations
- matrix representations of linear mappings
- eigenvalues and eigenvectors
- subspaces and (orthogonal) complements

Problems/challenges

- routine calculations can be done by WolframAlpha
  $\rightsquigarrow$ check more than the result (e.g. intermediate steps)
- tasks with many correct solutions (linear equations, bases of subspaces, etc.)

Topics in Multivariate Analysis

Examples

- multidimensional chain rule
- tangent space/normal space of submanifolds
- implicit function theorem
- local extrema with and without constraints

Observation

Small/innocent changes in parameters can render a given problem much harder or even untracktable.

(Trivial?) randomization tipps

1. Do not roll a dice for each parameter
2. Special matrices can be helpful
3. Even seemingly trivial operations can be useful

Randomization tipps

Do not roll a dice for each parameter

For a calculation with a plane

$$\vec{x} = \vec{a} + s \cdot \vec{b} + t \cdot \vec{c}$$

do not choose entries of $\vec{a}$, $\vec{b}$, $\vec{c}$ by random.

- $\vec{b}$ or $\vec{c}$ might happen to be $\vec{0}$
- $\vec{b}$ and $\vec{c}$ may turn out to be linearly dependent

Randomization tipps

Suggestions

- Use the full range of STACK randomization methods like
  - e.g. `rand_with_prohib` e.g. to prevent zeros
  - rand from a list for nonsystematic choice of values
- „Dirty“ mathematics is sometimes enough, e.g. use parity to ensure that two vectors are not linearly dependent
- random permutations
- `r:rand(1)` with `r*option1 + (1-r)*option2` can be used as a switch, e.g. transpose matrix or not

Randomization tipps

Special matrices can be helpful

Like before: „Dirty“ mathematical methods are ok (in my view)
For example:

- Need not be able to generate *all* invertible matrices
- instead: product of invertible lower and upper triangular matrix

Randomization tipps

A list of matrices with determinant ± 1 can be handy

Remember: For a square matrix A with integer entries and determinant ± 1, the inverse A^{-1} will also have integer entries.

$$A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 3 & -2 \\ 2 & 1 & 0 \end{pmatrix} \Rightarrow A^{-1} = \begin{pmatrix} 2 & 1 & -1 \\ -4 & -2 & 3 \\ -7 & -4 & 5 \end{pmatrix}$$

So $A^{-1}DA$ gives a matrix with integer entries and eigenvalues from diagonal matrix D.

Randomization tipps

Even seemingly trivial operations can be useful

Sometimes: difficult to generate „really different“ versions

Complete the following text:

The equation

$$3 \cdot \cos(3 \cdot x \cdot y) + y^3 + 3 \cdot x \cdot y - x^2 + 5 = 0$$

can be solved for y in a vicinity of the point $(x_0, y_0) = (0, -2)$ because the partial derivative of $f(x, y) = 3 \cdot \cos(3 \cdot x \cdot y) + y^3 + 3 \cdot x \cdot y - x^2 + 5$ with respect to [ ] in $(0, -2)$

takes the value [ ] $\neq 0$.

Writing the solution near $(x_0, y_0) = (0, -2)$ in the form $y = h(x)$, we arrive at the following Taylor polynomial of degree 2 for h:

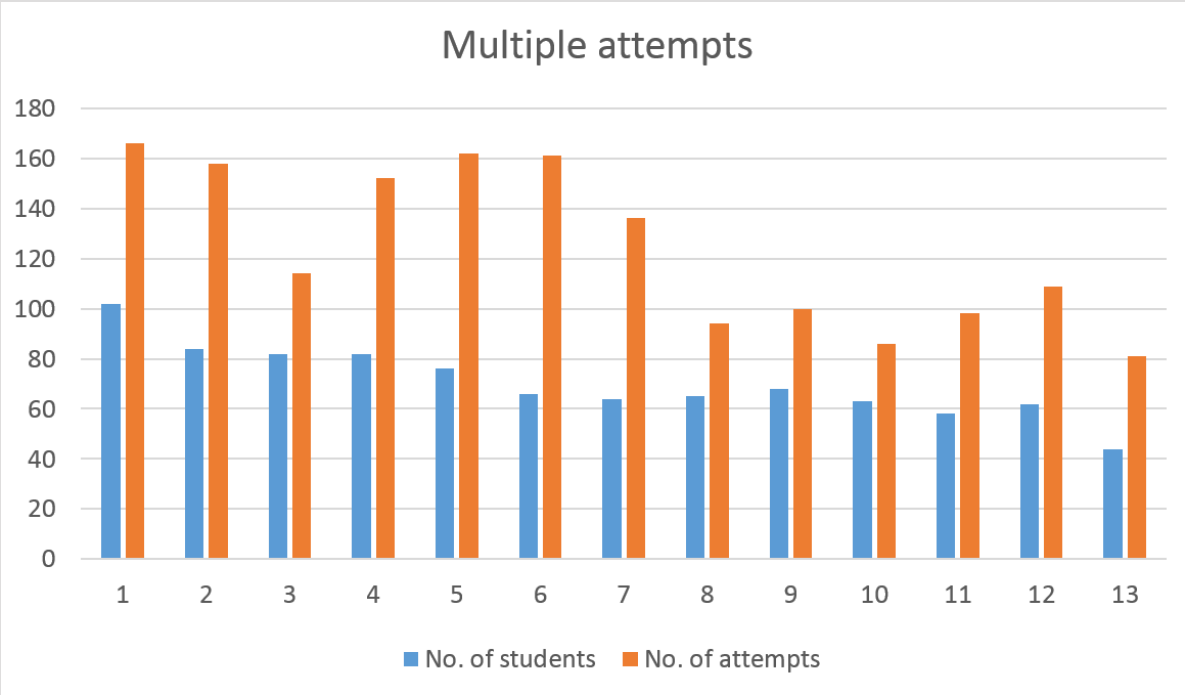
$h(x) =$ [ ] $+$ [ ] $(x) +$ [ ] $(x)^2$.

- name variables by random
- interchange x and y
- shift by some numbers
- (probably) nobody will notice the similarity

Some numbers and diagrams

- data from a course *Linear algebra for physicists* 2018
- each week one STACK exercise + three written homework exercises
- possibility to earn bonus points toward the exam

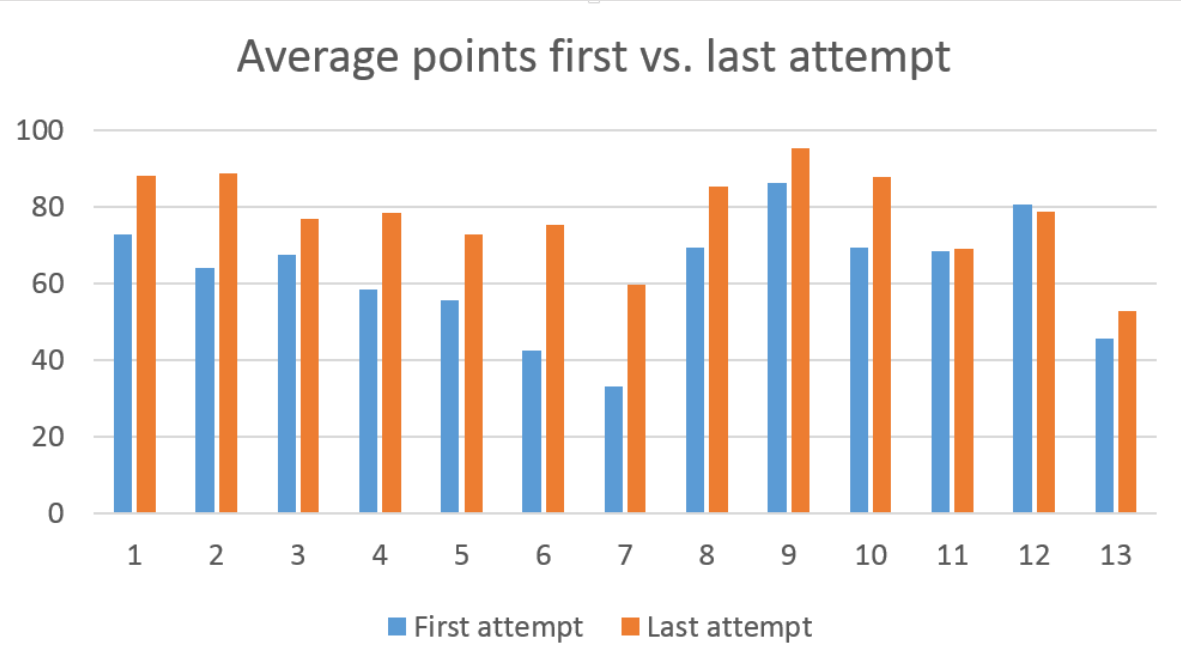
Multiple attemps



Observation

Students generously use the possibility to try the same exercise several times.

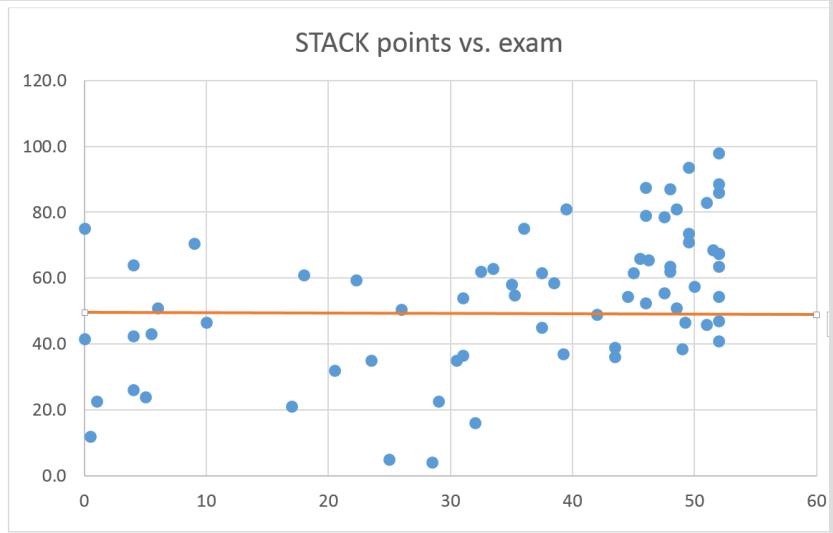
Results

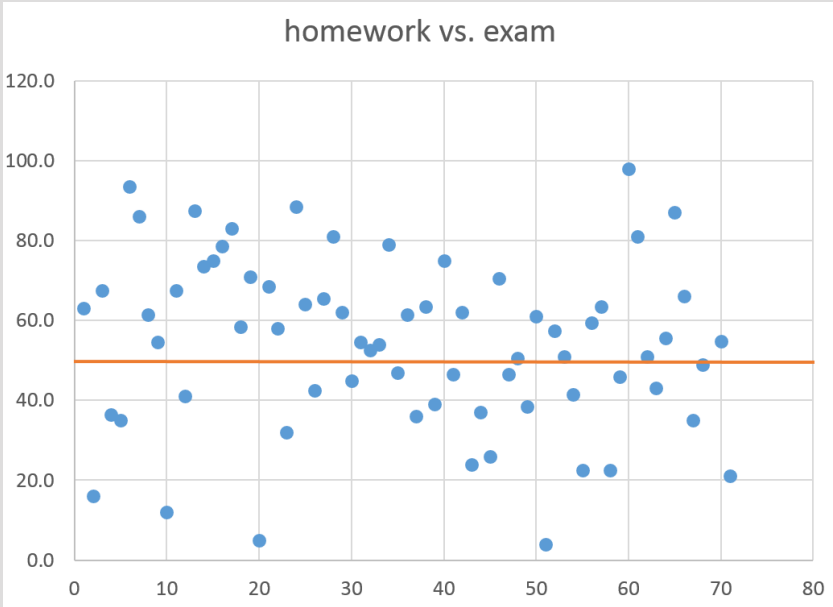


Observation

This allows most students to earn many bonus points by working on STACK exercises.

Results

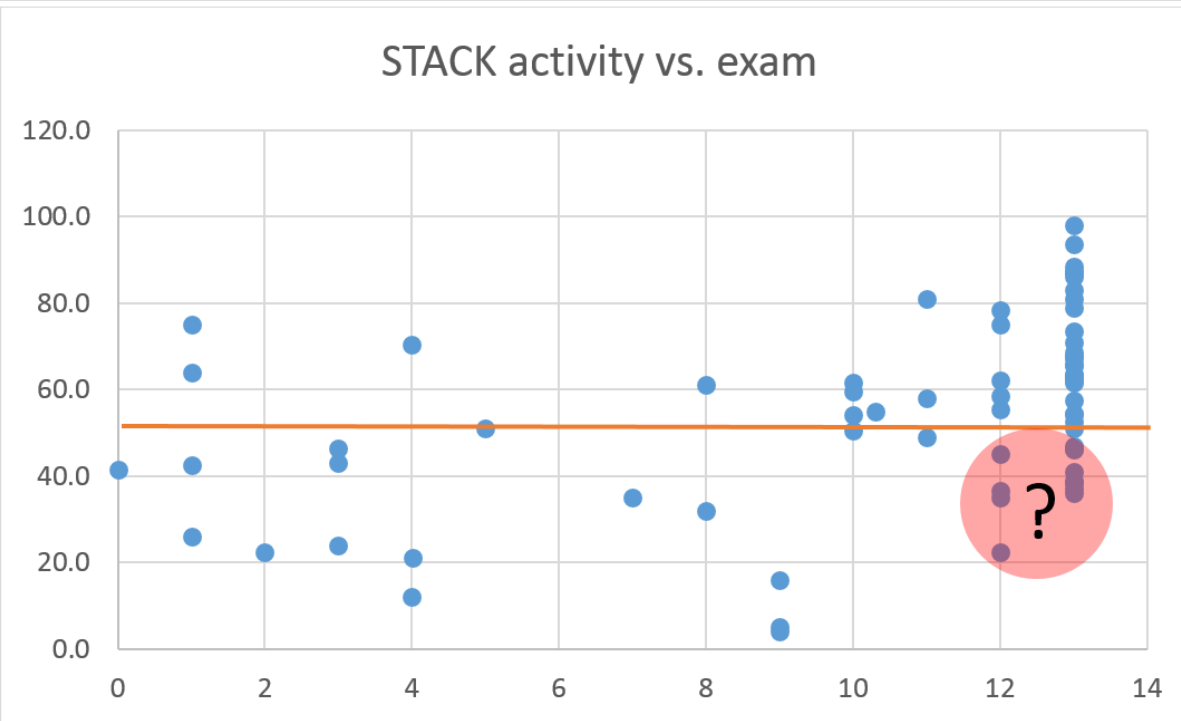




Observation

Correlation of STACK exercises much better than homework exercises.

Results



Observation

Looking at activity underscores the previous observation.