

Dynamic nature of mathematical concepts - experiences of learning materials in Moodle-Stack system

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2018, November

Briefly about our history

Our department "produces" mainly teachers for mathematical subjects, not much for technical professions. However, we want to offer solid mathematical background, and we also prepare 1-2 doctors per year in mathematics.

We try to enrich the learning materials by showing dynamic features of concepts, and nurture interactive ways to communicate with concepts and processes.

- ▶ interactive dynamic figures from year 2000 using Javasketchpad
- ▶ dynamic figures on html-forms, with open and MC-questions
- ▶ dynamic figures and questions in WebCT, and later to Moodle
- ▶ joined to ABACUS project in 2016; Stack-questions available
- ▶ Javasketchpad-to-JSXgraph converter 2017-2018 (Henri)

Some basic mathematical concept:

- ▶ relations, functions, abstract binary operations
- ▶ vectors, linear space, basis, coordinates, dimension
- ▶ linear functions and their matrix representations
- ▶ eigenvectors and eigenvalues in 2D plane

Concept representations - from concrete to abstraction

- ▶ verbal, symbolical and graphical (or other visual) expression
- ▶ in definitions, a verbal expression usually ties symbolical or verbal representations of other, former concepts
- ▶ matching between different representations of a concept examples
- ▶ transforming concept examples from one representation to another
- ▶ rich variety of representations

Our aims and objectives

To support first year students' abstraction processes using dynamic Moodle/STACK/JSXGraph questions, in addition to other Moodle question types.

- ▶ binary operation in action level: calculate
- ▶ match graphical and symbolical (IGS)
- ▶ recognize validity of definition requirements (DrG)
- ▶ reinforce derived concepts: image and pre-image
- ▶ solve "graphically": image-state answer
- ▶ solve "graphically": input numerical information
- ▶ linear space: recognition of basis
- ▶ relation: modify a relation to a transitive relation

Questions that request image-state answer

Using elements of dynamic images linked to the answer input or using question blocks

- ▶ point and it's coordinates or
- ▶ any data calculated from element(s) related to movable point(s)

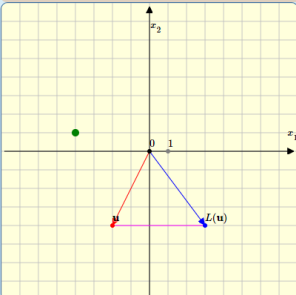
Improving automatic feedback:

- ▶ Including JavaScript (JS) into answer tree to complement JS in question text.
- ▶ Create new elements or make elements visible in original image.
- ▶ Build a new image with feedback.

Example

Kuva 1: Example of JSXGraph-STACK question with dynamic geometry, and feedback that is displayed for student.

We are looking at the function $L: \mathbb{R}^2 \rightarrow \mathbb{R}^2$, whose behaviour can be seen in a dynamic figure below. Tidy question | Suorita testitapauk



Your task is to set the variable in the figure so that it is the pre-image of the point $\begin{pmatrix} 6 \\ 1 \end{pmatrix}$.

Your answer is incorrect.

Alla olevat punaisella merkityt alkioit ovat väärin. $[-2, -4]$

The pre-image of point $(6, 1)$ is in point $(-4, 1)$, which is now placed in figure (green). You can check that this actually is the asked pre-image by dragging the variable to this point.

You set the variable to point $(-2, -4)$, which is not the pre-image of point $(6, 1)$.

Vastaa uudelleen

How to execute satisfying automatic assessment

- ▶ Binding points or other elements in JSXGraph image to STACK question's input field can be done flexible with JS.
- ▶ Input values can be more than just point coordinates.
- ▶ JavaScript/JSXGraph functions and methods like Dist and getSlope can do calculation and pre-checking for values to complement STACK-Maxima.
- ▶ Several elements info can be relayed to single input text field.

Example

Kuva 2: Example how to insert various data into input field. Here not only objects coordinates are inserted, but also the actual relation (which normally is hidden from the student) that is used to test the answer.

There is an arrow diagram of some relation R below. Your task is to modify the arrow diagram so that the corresponding new relation is both **symmetric** and **transitive**. Tidy question | Suorita testitapaukset...

In the modified relation the amount of ordered pairs must still be the same as it was in the original relation. So you can not just move some arrows or loops aside the figure or place them on top of each others to construct same ordered pairs multiple times, and every arrow endpoint and red loop point must be inside some gray disc (these represent the numbered elements).

Hint: Placing arrows and loops may be easier by dragging them first by their shaft aside from their endpoints. After this dragging the endpoints on top of the wanted elements is easier.

`[[[-3.13,16],[2.67,11.7]],[[2.2],[12.16,5.86],[10.81,5.21]],[[1,1],[12.16,-5.86],[10.81,-5.21]]],[[7.4],[6.1],[7.3],[5.3],[7.5],[1.2],[4.2],[1.4],[6.6],[5.5],[3.3],[2.2],[1.1]]]`

Lukitsen vastaukseni

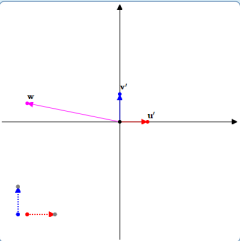
Example: Creating more useful feedback for students

Kuva 3: Adding new JSXGraph image with question variables and values as feedback.

In the figure below you can see a linear combination \mathbf{w} of vectors \mathbf{u} and \mathbf{v} so that $\mathbf{w} = 2\mathbf{u} - 3\mathbf{v}$.

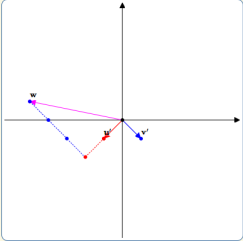
Your task is to solve vectors \mathbf{u} and \mathbf{v} . Place the heads of the vectors \mathbf{u}' and \mathbf{v}' to match the vectors \mathbf{u} and \mathbf{v} in the linear combination. The state in which you place the vectors \mathbf{u}' and \mathbf{v}' works as your answer to the question.

You may use movable help vectors at the bottom corner of the figure. Help vectors have the same direction and norm as vectors \mathbf{u}' and \mathbf{v}' . Help vectors can be dragged by their base point.



Lukitsen vastaukseni

Incorrect.
Your answer is incorrect. One solution for vectors \mathbf{u}' and \mathbf{v}' is shown in figure below.



Vastaa uudelleen

Creating more interactive questions

- ▶ Giving instant feedback while student(s) are solving the question.
- ▶ Linking answer field to JS in question text to create versatile syntax hints, which seems to be important for students with low programming background.
- ▶ Binding answer field to JSXGraph image elements to create discrete dynamic question environment
 - ▶ image itself is dynamic, but it is also malleable and can be updated by discrete user input in answer field

Example

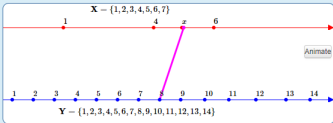
Kuva 4: Image in question includes dynamic geometry objects, and typing into the answer field inputs also updates the objects in the image.

Bijection

A bijection is a function that has following properties

1. distinct elements of the domain have distinct images in the codomain (injectivity),
2. each element of the codomain is the image of some element of the domain (surjectivity).

In the following figure you see the rule of a relation from \mathbf{X} to \mathbf{Y} by clicking the button Animate or moving the variable x by mouse. Notice that initially the rule **may not** define a function $\mathbf{X} \rightarrow \mathbf{Y}$.



a) Modify the domain \mathbf{X} to a set \mathbf{X}' so that there are as many elements as possible in \mathbf{X}' and the rule in the figure is an injection $\mathbf{X}' \rightarrow \mathbf{Y}$.

$\mathbf{X}' =$

Vastauksesi tulkittiin muodossa: {1, 4, 6}

b) Modify also the codomain to a set \mathbf{Y}' so that the rule is good for a bijection $\mathbf{X}' \rightarrow \mathbf{Y}'$.

$\mathbf{Y}' =$

Vastauksesi tulkittiin muodossa: { }

References

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